Parametric Resonance Amplification of Neutrino Oscillations in Electromagnetic Wave with Varying Amplitude and "Castle Wall" Magnetic Field

M.S. Dvornikov, A.I. Studenikin[1]

Department of Theoretical Physics, Moscow State University,

119899 Moscow,Russia

Abstract

Within the Lorentz invariant formalizm for description of neutrino evolution in electromagnetic fields and matter we consider neutrino spin oscillations in the electromagnetic wave with varying amplitude and in "castle wall" magnetic field. It is shown for the first time that the parametric resonances of neutrino oscillations in such systems can occur.

[1]E-mail: studenik@srdlan.npi.msu.su

Experimental studies of solar, atmospheric and reactor neutrinos over the past several years provide almost certain indications that neutrinos oscillate, have masses and mix. These new properties of neutrinos, if confirmed by better statistics in the proceeding and forthcomming experiments, will require a significant departure from the Standard Model. It is well known [1] that massive neutrinos can have nonvanishing magnetic moment. For example, in the Standard Model supplied with $SU(2)$-singlet right-handed neutrino the one-loop radiative correction generates neutrino magnetic moment which is proportional to neutrino mass. There are plenty of models [2] which predict much large magnetic moment for neutrinos.

In the light of new developments in neutrino physics, to get better understanding of the electromagnetic properties of neutrinos is an important task. In general, there are two aspects of this problem. The first one is connected with values of neutrino electromagnetic form - factors. The second aspect of the problem implies consideration of influence of external electromagnetic fields, which could be presented in various environments, on neutrino possessing non - vanishing electromagnetic moments. The most important of the latter are the magnetic and electric dipole moments.

The majority of the previously performed studies of neutrino conversions and oscillations in electromagnetic fields deal with the case of constant transversal magnetic field or constant transversal twisting magnetic fields (see references [1-18] of paper [3]). Recently we have developed [3-5] the Loretz invariant formalism for description of neutrino spin evolution that enables one to consider neutrino oscillations in the presence of an arbitrary electromagnetic fields. Within the proposed approach it becomes possible to study neutrino spin evolution in an electromagnetic wave and the new types of resonance in the neutrino oscillations in the wave field and in some other combinations of fields have been predicted. The latest application [6] of this formalism enables us to consider neutrino oscillations in electromagnetic fields in moving and polarised matter and show that matter effects on neutrino oscillations in the case of relativistic motion of matter could sufficiently depend on matter motion.

The aim of this paper is to continue the study of the neutrino spin oscillations in different electromagnetic fields and consider the cases of electromagnetic wave with periodically varying amplitude (see also [7]) and periodic "castle - wall" magnetic field. It will be shown that under certain conditions the enhancement of neutrino oscillations could be achieved due to the parametric resonance effect.

The possibility of the parametric resonance of neutrino flavour oscillations in matter with periodic variation of density was considered previously in [8]. It should be also pointed here that the conditions for a total neutrino flavour conversion in a medium consisting of two or three constant density layers were derived in [9]. For the recent discussion on the physical interpretations of these two mechanisms of increasing of neutrino conversion see refs.[9-11].

First we consider the case when the case of electromagnetic wave, with the amplitude of which being not constant but periodically varying in time, and show (see also ref.[7]) that under certain conditions the parametric resonance of neutrino oscillations can occur in such a system. Note that in order to investigate this phenomenon we have to use the Lorentz invariant formalisms for neutrino evolution that have been developed in [3-5].

Let us consider evolution of a system $\nu = (\nu_+, \nu_-)$ composed of two neutrinos of different helicities in presence of a field of circular polarised electromagnetic wave with periodically varying amplitude. Here neutrinos ν_+ and ν_- correspond to positive and negative helicity states that are determined by projection operators

$$P_{\pm} = \frac{1}{2}\left(1 \mp \frac{(\vec{\sigma}\vec{p})}{|\vec{p}|}\right). \tag{1}$$

In general case it is important to distinguish helicity states $\nu = (\nu_+, \nu_-)$ and chirality states $\nu = (\nu_L, \nu_R)$. The latter are determined by the projection operators $P_{L,R} = (1 \mp \gamma_5)/2$. The evolution of such a system is given [3-5] by the following Schrödinger type equation

$$i\frac{\partial \nu}{\partial t} = H\nu, \quad H = \tilde{\rho}\sigma_3 + E(t)(\sigma_1 \cos\psi - \sigma_2 \sin\psi), \tag{2}$$

$$\tilde{\rho} = -\frac{V_{eff}}{2} + \frac{\Delta m^2 A}{4E}, \quad E(t) = \mu B(t)(1 - \beta\cos\phi).$$

Here the three parameters, $A = A(\theta)$ being a function of vacuum mixing angle, $V = V(n_{eff})$ being the difference of neutrino effective potentials in matter, and Δm^2 being the neutrino masses squired difference depend on the considered type of neutrino conversion process. The electromagnetic field is determined in the laboratory frame of reference by its frequency, ω, the phase at the point of the neutrino location, $\psi = g\omega t(1 - \beta/\beta_0 \cos\phi)$,$(g = \pm 1)$, and the amplitude, $B(t)$, which is a function of time. The wave speed in matter could be less than the speed of light in vacuum ($\beta_0 \leq 1$), and ϕ is the angle between the neutrino speed $\vec{\beta}$ and the direction of the wave propagation. In the derivation of the Hamiltonian of eq.(2) terms proportional to $\frac{1}{\gamma^2} \ll 1$, $\gamma = (1-\beta^2)^{-1/2}$, and also an oscillating function of time in the diagonal part are neglected (for details see [3-5]).

In order to study phenomenon of the parametric resonance of neutrino spin oscillations in such a wave we suppose that the amplitude $B(t)$ is given by

$$B(t) = B(1 + hf(t)), \quad (|h| \ll 1), \tag{3}$$

where $f(t)$ is an arbitrary function of time and h is a small dimensionless quantity of not fixed sign. It is convenient to introduce the evolution operator which determines the neutrino state at time t

$$\nu(t) = U(t)\nu(0)$$

if the initial neutrino state was $\nu(0)$. Using the Hamiltonian of eq.(2) we get the following equation for the evolution operator:

$$\dot{U}(t) = i[-\tilde{\rho}\sigma_3 + (-E_0 + \varepsilon f(t))(\sigma_1 \cos\psi - \sigma_2 \sin\psi)]U(t), \tag{4}$$

where c$\varepsilon = -E_0 h$. In analogy with the case of the electromagnetic wave with non - varying amplitude [3,4] the solution of eq.(4) can be written in the form

$$U(t) = U_{\vec{e}_3}(t) U_{\vec{l}}(t) F(t), \tag{5}$$

where $U_{\vec{e}_3}(t) = \exp(i\sigma_3 \frac{\dot{\psi}t}{2})$ is the rotation operator around the axis $\vec{e}_3$ which is parallel with the direction of the neutrino propagation, and $U_{\vec{l}}(t) = \exp(i\vec{\sigma}\vec{l}t)$ is the rotation operator around the vector $\vec{l} = (-E_0, 0, \tilde{\rho} - \frac{\dot{\psi}}{2})$. It should be noted that the solution of eq.(4) for the case of constant amplitude of the wave field ($\varepsilon = 0$) is given by the operator $U_0(t) = U_{\vec{e}_3}(t) U_{\vec{l}}(t)$.

From (4) and (5) it follows that the equation for the operator $F(t)$ is

$$\dot{F}(t) = i\varepsilon H_\varepsilon(t) F(t), \tag{6}$$

where

$$H_\varepsilon(t) = (\vec{\sigma}\vec{y}(t)) f(t),$$

$$\vec{y} = (1 - 2\lambda_3^2 \sin^2 \Omega t,\ \lambda_3 \sin 2\Omega t,\ 2\lambda_1\lambda_3 \sin^2 \Omega t),$$

and the unit vector λ is given by its components in the unit orthogonal basis $\vec{\lambda} = \lambda_1 \vec{e}_1 + \lambda_2 \vec{e}_2 + \lambda_3 \vec{e}_3 = \frac{\vec{l}}{\Omega}, \Omega = |\vec{l}|$. The detailed analysis of evolution of the solution of eq.(6) can be found in [12]. Here we comment only on the main steps. Using the smallness of ε we expand the solution of eq.(6) in powers of this parameter:

$$F = \sum_{k=0}^{\infty} \varepsilon^k F^{(k)}, \tag{7}$$

where $F^{(0)} = \hat{1}$ is a unit matrix. For operators $F^{(k)}$ the recurrent formula is straightforward:

$$F^{(k+1)}(t) = i \int_0^t H_\varepsilon(\tau) F^{(k)}(\tau) d\tau. \tag{8}$$

Skipping further technical details to the first order in ε we get

$$F(t) = \hat{1} + i\varepsilon(\vec{\sigma}\vec{x}(t)) + O(\varepsilon^2), \tag{9}$$

where

$$\vec{x}(t) = \int_0^t \vec{y}(\tau) f(\tau) d\tau.$$

Thus for the probability of neutrino conversion $\nu_{i-} \leftrightarrow \nu_{j+}$ we get

$$P_{ij} = | < \nu_+ | U_{\vec{e}_3}(t) U_{\vec{l}}(t) F(t) | \nu_- > |^2 = \tag{10}$$

$$\lambda_1^2 \sin^2 \Omega t + 2\varepsilon\lambda_1 (x_1(t) \cos \Omega t + \lambda_3 x_2(t) \sin \Omega t) \sin \Omega t.$$

Note that Ω is nothing but the mean osillation frequency of the neutrino system.

For further evaluation of solution of eq.(6) we have to specify the form of the function $f(t)$. As it is mentioned above, the purpose of this study is to examine the case when the parametric resonance for neutrino oscillations in electromagnetic wave with varying amplitude could appear. Having in mind a simple analogy (see, for example, [10]) between oscillations in neutrino system and oscillations of a classical pendulum [13], it is reasonable to suppose that the principal parametric resonance appears when the amplitude modulation function $f(t)$ is oscillating in time with frequency approximately equal to the twice mean oscillation frequency of the system. That is why we choose the function $f(t)$ to be

$$f(t) = \sin 2\Omega t \tag{11}$$

and get for the neutrino oscillations probability

$$P_{ij} = [\lambda_1^2 + \varepsilon\lambda_1\lambda_3^2 t + \frac{\varepsilon\lambda_1}{\Omega}(1 - \frac{\lambda_3^2}{2}) \sin 2\Omega t] \sin^2 \Omega t. \tag{12}$$

It follows that in the case $\lambda_1\varepsilon > 0$ the second term increases with increase of time t, so that the amplitude of the neutrino conversion probability may becomes close to unity. This is the effect of the parametric resonance in neutrino system in the electromagnetic wave with modulated amplitude that may enhance neutrino oscillation amplitude even for rather small values of the neutrino magnetic moment μ and strength of the electromagnetic field and also when the system is far away from the region of ordinary spin (or spin-flavour) neutrino resonance.

Let us consider the case when parameters of the neutrino system are far beyond the region of the ordinary neutrino resonance. Supposing that $|l_1| = 0.1|l_3|$. In this case the maximal neutrino conversion probability, for non - varying ($h = 0$) amplitude of the electromagnetic field, is indeed small,

$$P_{ijmax}(h = 0) = \frac{l_1^2}{l_1^2 + l_3^2} \ll 1. \tag{13}$$

If the amplitude of the field is modulated in accordance with eq.(11) the estimation for the critical time t_{cr} for which the probability P_{ij} could become close to unity for an arbitrary values of the neutrino mixing angle, masses, energy, and density of matter gives

$$t_{cr} \approx \frac{1}{\varepsilon n_1}. \tag{14}$$

It means that the parametric resonance enhancement of the neutrino oscillations occurs after neutrinos travel a distance under the influence of electromagnetic wave with modulated amplitude. For the further numerical estimates me chose the strength of the field $B \approx 10^3$ G, the value of the neutrino magnetic moment $\mu \approx 10^{-10}\mu_B$ (μ_B is the Borh magneton), and also suppose that neutrino is propagating in the direction opposite to one of the electromagnetic wave. Thus, from (14) we obtain, for the small time variations of the field amplitude (taking, for instance $|h| = 0.1$), that the conversion probability could attain the value $P_{ij} \approx 1$ after the neutrino has travelled a distance

$$x_{min} = 10^{-12} m. \tag{15}$$

Let us consider now a system of two helicity neutrino states with mixing $\nu = (\nu_+, \nu_-)$. In general the neutrino components can belong to different flavours. The evolution of such a system in matter under the influence of transversal magnetic field is described by the Schrödinger equation:

$$i\frac{\partial \nu}{\partial t} = H\nu \tag{16}$$

with Hamiltonian

$$H = -\mu B(t)\sigma_1 + \tilde{\rho}\sigma_3, \tilde{\rho} = \frac{\Delta m^2}{4E}A - \frac{V_{eff}}{2} \tag{17}$$

Now we are going to discuss a possibility of parametric resonance amplification that could exist under the influence of constant in time but periodically varying in space magnetic field. We consider a special case when the strength of the field varies along the neutrino path and the dependence on the neutrino coordinate t is given by,

$$B(t) = \begin{cases} B_1, & 0 \le t < T_1 \\ B_2, & T_1 \le t < T_1 + T_2 \end{cases} \tag{18}$$

here B_1 and B_2 are constants. The field strength is periodic "castle wall" function with period $T = T_1 + T_2$: $B(t+T) = B(t)$. The effective Hamiltonian is also a periodic function of time with the same period T:

$$H(t+T) = H(t), \tag{19}$$

and

$$H(t) = \begin{cases} H_1, & 0 \le t < T_1 \\ H_2, & T_1 \le t < T_1 + T_2 \end{cases} \tag{20}$$

H_1, H_2 are the constant operators. It should be noted that the similar approach to neutrino oscillations in the case of periodic step - function ("castle wall") density was described in details in the first paper of ref.[10]

Let us define the evolution operators for the two time intervals $(0, T_1)$ and (T_1, T):

$$U_a = \exp(-iH_a T_a), \quad a = 1, 2. \tag{21}$$

Then the operator giving the neutrino evolution for the whole period T is

$$U_T = U_2 U_1. \tag{22}$$

For the further consideration it is convenient to introduce the unit vectors

$$\vec{n}_a = -\frac{1}{\omega_a}(E_a, 0, -\tilde{\rho}) = (\sin 2\theta_a, 0, -\cos 2\theta_a), \quad a = 1, 2, \tag{23}$$

where $E_a = \mu B_a$, $\omega_a = \sqrt{\tilde{\rho}^2 + E_a^2}$ and θ_a are the effective neutrino mixing angles accounting for interaction with matter and magnetic field. Then for effective Hamiltonians we can write

$$H_a = \omega_a(\vec{\sigma} \cdot \vec{n}_a), \tag{24}$$

As it is easy to see, the eigenvalues of H_a are $\pm\omega_a$. Using eqs.(18-24) we get the followig expressions for the neutrino evolution operator over the whole period T:

$$U_T = Y - i(\vec{\sigma}\cdot\vec{X}) = \exp(-i(\vec{\sigma}\cdot\vec{n}_X)\Phi), \tag{25}$$

where

$$Y = c_1c_2 - (\vec{n}_1\cdot\vec{n}_2)s_1s_2, \tag{26}$$

$$\vec{X} = s_1c_2\vec{n}_1 + s_2c_1\vec{n}_2 - (\vec{n}_1\times\vec{n}_2)s_1s_2, \tag{27}$$

$$\Phi = \arcsin Y = \arccos X, \tag{28}$$

$$\vec{n}_X = \frac{\vec{X}}{X}, \quad X = |\vec{X}|. \tag{29}$$

and

$$s_a = \sin\phi_a, \quad c_a = \cos\phi_a, \quad \phi_a = \omega_a T_a, \quad a = 1,2. \tag{30}$$

From (26),(27) it follows that

$$\vec{n}_1\vec{n}_2 = \frac{1}{\omega_1\omega_2}(B_1B_2 + \tilde{\rho}^2) = \cos 2(\theta_1 - \theta_2), \tag{31}$$

$$\vec{n}_1\times\vec{n}_2 = \frac{1}{\omega_1\omega_2}(0, \tilde{\rho}(B_1 - B_2), 0) = (0, \sin 2(\theta_1 - \theta_2), 0). \tag{32}$$

It should be noted also that $Y^2 + \vec{X}^2 = 1$ due to unitarity of the operator U_T. The vector $\vec{X}$ can be expressed in its components as

$$\vec{X} = \Big(- \frac{s_1c_2E_1}{\omega_1} - \frac{s_2c_1E_2}{\omega_2}, \tilde{\rho}\frac{s_1s_2}{\omega_1\omega_2}(E_2 - E_1), \tilde{\rho}(\frac{s_1c_2}{\omega_1} + \frac{s_2c_1}{\omega_2})\Big). \tag{33}$$

Let us consider the case when neutrino path in matter and periodic "castle wall" magnetic field is exactly equal to a whole number of periods, so that $t = nT, n \in N$. The neutrino evolution operator for n periods is equal to U_T in the nth power. Assume that initially the two - level neutrino system concerned occupies the state ν_-.

$$U_{nT} = \exp(-i(\vec{\sigma}\cdot\vec{n}_X)n\Phi). \tag{34}$$

Assume that initially ($t = 0$) the two - level neutrino system involved occupies the state ν_-. The transition probability to the state ν_+ after a time $t > 0$ is determined by the evolution operator U_T:

$$P(t) = | < \nu_+ | U(t) | \nu_- > |^2. \tag{35}$$

For the case $t = nT$ using eqs.(34),(35) we get

$$P(nT) = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2(n\Phi) = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2(\Phi \frac{t}{T}). \tag{36}$$

This expression coincides with one obtained in the first paper of ref.[10] for neutrino oscillation probability in step - function profile of matter density. However, it is obvious that in our case X_i and Φ have different meaning.

Expression (36) also looks like the neutrino oscillations probability for the case of not varying in space (non-periodic) magnetic field. However, there is an important distinction from the case of non-periodic magnetic field. The pre-sine factor in the latter case equals $\frac{\mu B^2}{\mu B^2 + \tilde{\rho}^2}$ which is small if the value μB is rather small. Contrary to the case of non-periodic magnetic field, in the case of $B_1 \neq B_2$ the pre-sine factor need not be small even if $\sin^2 2\theta_a$ are small. For particular choice of parameters the pre-sine term in eq.(36) becomes equal to unity. It is this case which we call the parametric enhancement of neutrino oscillations in periodic "castle wall" magnetic field. The resonance condition is

$$X_3^2 = \tilde{\rho}^2 (\frac{s_1 c_2}{\omega_1} + \frac{s_2 c_1}{\omega_2})^2 = 0, \tag{37}$$

Now let us consider the case when the neutrino oscillations can be considerably increased due to the effect of parametric resonance in "castle wall" magnetic field. Assume that the magnetic field is rather small.

$$(\mu B_a)^2 = E_a^2 \ll \Delta^2, \tag{38}$$

Then from the resonance condition of eq.(37) it follows that

$$\phi_1 + \phi_2 = \pi k, \quad k \in N. \tag{39}$$

If we introduce the mean oscillation frequency as

$$\bar{\omega} = \frac{\omega_1 T_1 + \omega_2 T_2}{T}, \tag{40}$$

then the resonance condition can be rewritten in the form

$$\Omega = \frac{2\bar{\omega}}{k}, \tag{41}$$

where $\Omega = \frac{2\pi}{T}$ is the magnetic field variation frequency ($T = T_1 + T_2$). The last formula again reveals the well known feature of the parametric resonance in an oscillating system: the resonance occurs when the doubled frequency, $2\bar{\omega}$, is equal to the frequency of a parameter variation multiplied by an integer number.

Consider the transition probability, eq.(36), in the case of the parametric resonance (39). Thus from (33) for this case we get

$$\vec{X}^2 = \frac{1}{\tilde{\rho}^2}(s_1^2 E_1^2 + s_2^2 E_2^2 + 2 s_1 E_1 s_2 E_2 (-1)^k). \tag{42}$$

Following the analysis performed in the first paper of ref.[10], one can show that the optimal conditions for having transition probability are realised when

$$\phi_a = \frac{\pi}{2} + \pi k_a, \quad a = 1, 2, \tag{43}$$

and $k_1 + k_2 \geq 0$. Accounting (38) and (43) from (42) it follows that

$$|\vec{X}| = \left| \frac{E_1 - E_2}{\tilde{\rho}} \right| \ll 1. \tag{44}$$

Thus, for the transition probability (36) in the case of parametric resonance we get

$$P(nT) = \sin^2 \left(n \frac{(E_1 - E_2)}{\tilde{\rho}} \right) = \sin^2(2n(\theta_1 - \theta_2)). \tag{45}$$

In derivation of the last expression we use the conditions $\theta_a \ll 1$. The value of $\tilde{\rho}$ being equal to $\frac{\pi k}{T}$, the formula (45) can be rewritten in the form

$$P(nT) = \sin^2 \left(\frac{(E_1 - E_2)}{\pi k} t \right) \tag{46}$$

It follows that the parametric enhancement of neutrino oscillations becomes most sufficient when $k = 1$. The similar effect exists for the parametric resonance in mechanical oscillating system that underlines the analogy mentioned above.

Let us perform numerical estimates of possible effect of the parametric resonance amplification of neutrino oscillations in "castle - wall" magnetic field. We assume for simplicity that $T_1 = T_2 = D$ and neglect matter effects. From the resonance condition (41) follows that

$$D = \frac{2\pi k E}{\Delta m^2 A}. \tag{47}$$

To obtain a maximal effect we chose $k = 1$. Then, for the particular choice of the neutrino characteristics $\Delta m^2 = 0.1eV^2$, $E = 0.1eV$, $\theta_{vac} = 0$, we get $D = 1m$. If the magnetic field profile corresponds to a chain of solenoids with opposite direction of electric currents, that results in $B_1 = -B_2 = B$ in terms of magnetic field direction, then for the oscillation probability we get

$$P(nT) = \sin^2(4n\theta), \tag{48}$$

where $2\theta = \mu B/(\frac{\Delta m^2}{4E})$ For the neutrino transition magnetic moment $\mu = 10^{-10}\mu_B$ and the strength of magnetic field $B = 10^7$ G, we obtain $2\theta \approx 2.3 \cdot 10^{-5}$. Thus, the probability is given by

$$P(nT) = \sin^2(4.6\dot{1}0^{-5}n), \tag{49}$$

The probability attains the value $P(nT) \approx 0.1$ (10% of the total number of neutrinos are converted) when the number of periods equals to 7000.

The authors are thankful to Evgeny Akhmedov and Sergey Petcov for useful discussions. One of us (A.S.) should like to thank Mario Greco for invitation to Valee d'Aoste and to thank all the organizers of this important meeting for hospitality.

References

[1] B.Lee, R.Shrock, *Phys.Rev.*D **16**, 1444 (1977); K.Fujikawa, R.Shrock, *Phys.Rev.Lett.* **45**, 963 (1980).

[2] J.Kim, *Phys.Rev.*D **14**, 3000 (1976); M.Beg, W.Marciano, *Phys.Rev.*D **17**, 1395 (1978); M.Voloshin, M.Vysotsky, L.Okun, *JETP* **64**, 446 (1986); M.Fukugita, T.Yanagida, *Phys.Rev.Lett.* **58**, 1807 (1987).

[3] A.Egorov, A.Lobanov, A.Studenikin, *Phys.Lett.*B **491**, 137 (2000); hep-ph/9910476.

[4] A.Egorov, A.Lobanov, A.Studenikin, in : A.Mourao, M.Pimento, P.Sa (Eds.), New Worlds in Astroparticle Physics, World Scientific, Singapore, 1999, p.153; hep-ph/9902447.

[5] A.Egorov, A.Lobanov, A.Studenikin, in : M.Greco (Ed.), Results and Perspective in Particle Physics, Frascaty Physics Series, v. XVII, p.117, 2000.

[6] A.Lobanov, A.Studenikin, Phys.Lett.B (2001), accepted for publication; hep-ph/0106101.

[7] M.Dvornikov, A.Studenikin, in : P.Sa (Ed.), New Worlds in Astroparticle Physics, World Scientific, Singapore, 2001; hep-ph/0102099.

[8] V.Ermilova, V.Tsarev, A.Chechin, Short Notices of the Lebedev Institute, v. 5, p. 26, 1986; E.Akhmedov, SJNP **47**, 301 (1988); P.Krastev, A.Smirnov, PLB **226**, 341 (1989).

[9] S.Petcov, *Phys.Lett.*B **434**, 321 (1998); (E) *Phys.Lett.*B **444**, 584 (1998); M.Chizov, S.Petcov, *Phys.Rev.Lett.* **83**, 1096 (1999); hep-ph/9903424; *Phys.Rev.*D **63**, 073003 (2001) .

[10] E.Akhmedov, *Nucl.Phys.*B **538**, 25 (1999); E.Akhmedov, A.Smirnov, hep-ph/9910433.

[11] M.Chizov, S.Petcov, hep-ph/0003110.

[12] M.Dvornikov, A.Studenikin, *Phys.At.Nucl.* **64**, 1 (2001).

[13] L.Landau, E.Lifshits, Course of Theoretical Physics, V.1, Mechanics, 3rd ed., Pergamon Press, London, 1976.